

# Modelling Share Price Behaviour Across Time

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## Abstract

The Efficient Markets Hypothesis (EMH) is currently the dominant paradigm in Finance. This paper reviews the theoretical development of the hypothesis and the empirical testing which has occurred to determine its validity. Furthermore, empirical anomalies found by researchers in the Weak Form of the EMH are discussed and their theoretical interpretation critiqued. This paper also provides an overview of the Hamilton (1989) model and its extensions, one of the many econometric models developed in order to model the non-linearity in time-series such as stock prices.

## Modelling Share Price Behaviour Across Time

### 1. Introduction

Although there has been interest in the behaviour of share prices and returns for sometime, it was not until the late 1960s and early 1970s that a fully-developed, empirically-supported theory of share behaviour emerged in the form of the Efficient Markets Hypothesis (EMH). Prior to the development of the EMH in full, market participants assumed implicitly some degree of dependence across successful price changes. Much energy was devoted towards identifying a predictable trading pattern which could be used for profitable purposes. From the mid-1950s to the early 1980s, a random walk theory of share prices developed based on the accumulated empirical evidence of randomness in share price movements.

The mathematical theory of random speculative prices was pioneered by Bachelier in 1900. It basically said that speculative price changes were independent and identically distributed, so that the past price sequence had no predictive power for future price outcomes. Furthermore, it said that the distribution of price changes from transaction to transaction had finite variance. In formal terms:

$$p_t - p_{t-1} = \varepsilon_t \quad (1)$$

where  $p_t$  and  $p_{t-1}$  are the current period's and the previous period's price respectively, and  $\varepsilon_t$  is independently and identically distributed white noise. Alternatively, the probability distribution of the price of the share in period  $t + \tau$ , conditional on the set of information up to time  $t$ , is equivalent to its marginal probability distribution:

$$f(p_{t+\tau} / \phi_t) = f(p_{t+\tau}). \quad (2)$$

In addition, if transactions are fairly uniformly spread across time, and if the number of transactions is very large, then the Central Limit Theorem suggests that the price changes will be normally distributed.

This groundbreaking work went unrecognised by financial economists until the 1950's when evidence of randomness began to appear. Kendall (1953) calculated the first differences of twenty-two different speculative price series at weekly intervals ranging from 486 to 2,387 terms. Broadly speaking, he concluded that the random changes from one term to the next are so large as to swamp any systematic effect which may be present. In fact, Kendall (1953: 11) stated that 'the data behave almost like a wandering series'. Specifically, an analysis of share price movement revealed little serial correlation, with the conclusion that there was very little predicability of movements in share prices for a week ahead without extraneous information. Furthermore, Kendall found that although the distribution of price changes was leptokurtic, with too many values near the mean and too many values in the extreme tails, it was still approximately normally distributed. In 1959, Roberts elucidated that the intense interest in technical analysis was due to the fact that the usual method of plotting successive levels of share prices rather than changes gave the appearance of a pattern or trend in the data. He estimated the probability of different share price outcomes over time by using a frequency distribution of historical changes in the weekly market index, and assumed weekly changes were independently drawn from a normal distribution with a mean of + 0.5 and a standard deviation of 5.0. With this simple chance model, Roberts generated a pattern of market levels and changes incredibly similar to that of actual levels and changes in the Dow Jones Industrial Index. He therefore suggested that changes in security prices behave nearly as if they had been generated by a simple chance model which insists on independence but

makes no commitment about relative probabilities of different outcomes, except that they be stable over time.

The basic proposition behind the random walk theory is summarised by Cootner (1964). The fundamental concept is that competition in perfect markets would remove excess economic profits, except from those parties who exercised some degree of market monopoly. This meant that a trader with specialised information about future events could profit from the monopolistic access to information, but that fundamental and technical analysts who rely on past information should not expect to reap excess returns. Thus, changes in share prices could just as well be determined by a flick of a coin as by any sophisticated trading system or thorough analysis of past statistical information.

## **2. Efficient Markets**

From the empirical evidence and theory of random walks arose the theory of efficient markets. Fama (1970, 1976) provides a comprehensive survey of the early literature on both the theoretical and empirical aspects of the Efficient Markets Hypothesis, whilst Cuthbertson (1996) summarises the latest research developments.

### **Efficient Markets: Theory**

The Efficient Markets Hypothesis (EMH) states that current prices always 'fully reflect' available information, so that the only reason prices change between time  $t$  and time  $t+1$  is the arrival of 'news' or unanticipated events. The EMH is based on the assumptions of zero transaction costs, freely available information and agreement among investors on the implications of information on the share price. As Fama (1970) notes, these conditions do not hold in the real world. It is not necessary though for each, or all, of these assumptions to hold for the EMH to remain true. For example, the market can still be efficient if an adequately large number of traders have access to the necessary information. Thus, whilst these conditions are sufficient, they are not necessary.

The EMH requires that only two necessary conditions be met. First, the market must be aware of all available information. Formally stated, this means that the information set used by the market in time  $t$  to determine the price of security at time  $t$  ( $\phi_t^m$ ) is equivalent to the true information set ( $\phi_t$ ). The type of information contained in the information set is determined by the strength of the EMH being tested. In a Weak Form efficient market, current prices fully reflect what is knowable from the study of historical prices and trading volumes. Thus,  $\phi_t$  will contain the sequence of historical prices ( $p_t, p_{t-1}, p_{t-2}, \dots$ ). If the Weak Form is valid, technical analysis becomes ineffective. Any information contained in past prices has been analysed and acted on by the market, so that shares are neither under-valued nor over-valued. In a Semi-Strong Form efficient market, current prices efficiently adjust to information that is publicly available. Therefore,  $\phi_t$  will also contain publicly available information, such as earnings announcements, investments, dividends and capitalisation changes. If this form of the hypothesis holds true, then fundamental as well as technical analysis is also ineffective because all publicly available information has been thoroughly analysed, assessed and acted on by a vast number of analysts. Finally, in a Strong Form efficient market, current prices fully reflect all information, not just that included in the historical trading pattern or available through publicly released statements. Thus, if the Strong Form holds true, any attempt to make profitable use of

monopolistic access to information is useless because this information has already been incorporated into the market price of the share.

The second necessary condition states that the market correctly uses the available information in assessing the expected return of the share in the future period. Formally:

$$E(R_{t+1} / \phi_t) = \frac{E(P_{t+1} / \phi_t) - p_t}{p_t} \quad (3)$$

where  $E$  is the expected value operator,  $R_{t+1}$  is the return on the asset over period  $t+1$ , and  $P_{t+1}$  and  $p_t$  are the prices of the asset in period  $t+1$  and  $t$  respectively<sup>1</sup>. Alternatively, it may be represented as:

$$r_{t+1} = E(R_{t+1} / \phi_t) + \varepsilon_{t+1} \quad (4)$$

where  $E(\varepsilon_{t+1} / \phi_t) = 0$ . This second necessary condition is often referred to as the rational expectations element of the EMH, or informational efficiency. It means that actual returns can be randomly greater or lesser than expected returns, but on average, unexpected returns must be zero. The significant implication of the rational expectations element is that no system of trading rules can have greater expected returns than the equilibrium expected returns derived by the market. In other words, the hypothesis can be interpreted as a 'fair game' with respect to the information set

due to the fact that expected excess returns are zero. The orthogonality property states that  $\varepsilon_{t+1}$  must be independent of any information available at time  $t$  or earlier. If the error term is serially correlated, then the orthogonality property is violated. An example of a serially correlated error term is a first-order Markov scheme:

$$\varepsilon_{t+1} = r_{t+1} - E(R_{t+1} / \phi_t) = \rho\varepsilon_t + v_{t+1} \quad (5)$$

where  $\rho$  the first-order autocorrelation coefficient which falls between the value of  $-1$  and  $+1$ , and where  $v_{t+1}$  is the white noise error term, assumed independent of  $\phi_t$ . As the forecast error in time  $t$  is known, it thereby forms part of  $\phi_t$ , and has a predictable effect on the forecast error in time  $t+1$ . It would thereby violate the EMH as the forecast error can be used to predict future returns.

In essence, any test of the EMH is a joint test of firstly, whether the market makes efficient use of the information contained in the information set (the 'fair game' property), and secondly, of the market equilibrium expected return model incorporated into the hypothesised model. There are two important models of expected returns used in the early testing of the EMH. The first type tested was the constant expected returns model. An interpretation of the EMH is that a share's price represents the rational assessment by the market of fundamental value. Thus, the theory of share price asserts that a price is comprised wholly of a permanent or fundamental component. However, this relationship may be modelled mathematically in a number of different forms depending upon the assumptions made in relation to the intertemporal behaviour of fundamental value.

The permanent component may be represented by a model which assumes that the behaviour across time may be represented as a random walk with drift so that it follows that

<sup>1</sup> Throughout this paper, capital letters indicate a random variable and lower case letters indicate a particular realisation.

$$\begin{aligned}\ln p_{t+1} &= \textit{permanent} \\ &= \ln p_t + \delta + \varepsilon_{t+1}\end{aligned}\tag{6}$$

where  $\ln p_{t+1}$  is the natural log of the share price,  $\delta$  is the drift term and  $\varepsilon_t$  is an independent and identically distributed white noise error term. As the difference between the log of share prices is the actual continuously compounded return over that period:

$$r_{t+1} = \ln p_{t+1} - \ln p_t\tag{7}$$

then in terms of the random walk with drift model, actual continuously compounded returns over that period are equivalent to the drift term ( $\delta$ ) plus the error term ( $\varepsilon_t$ ):

$$r_{t+1} = \ln p_{t+1} - \ln p_t = \delta + \varepsilon_{t+1}.\tag{8}$$

If we assume rational expectations so that  $E(\varepsilon_{t+1} / \phi_t) = 0$ , then:

$$E(R_{t+1} / \phi_t) = \delta.\tag{9}$$

The unbiased estimator of expected returns  $E(R_{t+1} / \phi_t)$  is equivalent to the drift term  $\delta$ . Thus, returns over all periods are expected to be constant because the drift term  $\delta$  is independent of time and the information set.

Cuthbertson (1996) discusses how share prices are derived in an efficient market with constant expected returns. With some assumptions<sup>2</sup>, the price of a share will equate to the present value of expected future dividends based on the information set:

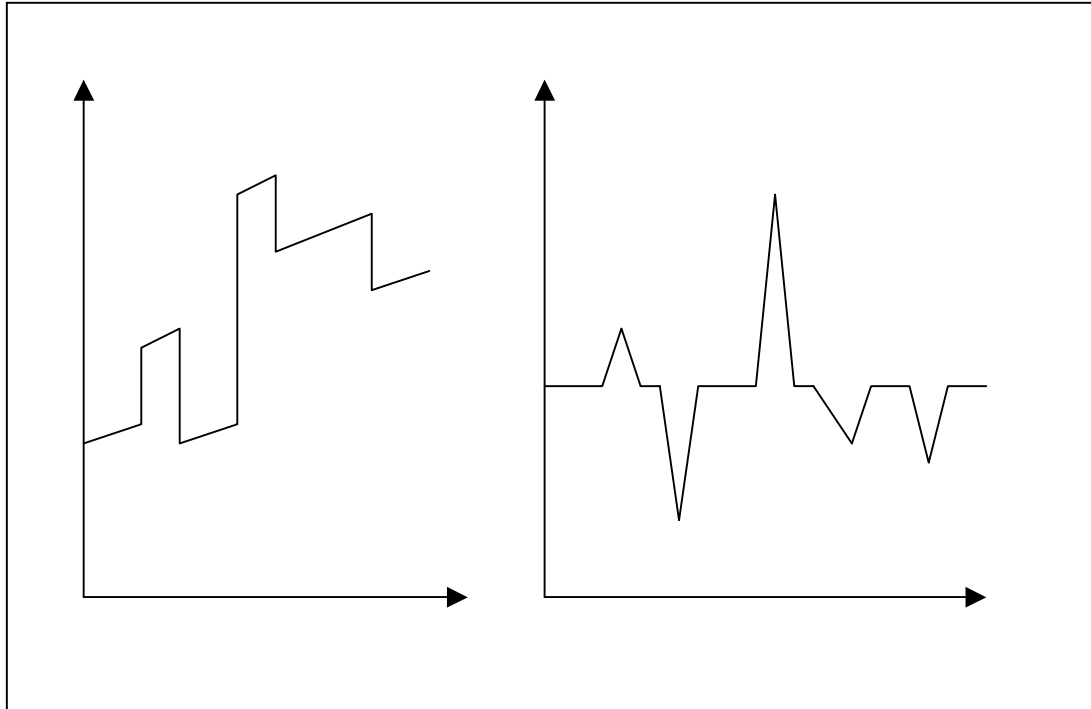
$$p_t = E\left[\sum_{i=1}^{\infty} \frac{D_{t+i}}{(1+\delta)^i} \middle| \phi_t\right]\tag{10}$$

where  $i = 0, 1, 2, \dots, \infty$  and  $D_{t+i}$  is current and future dividends. Thus, in an efficient market with constant expected returns, prices change because of fluctuations in expected fundamentals reflected in changing expectations of future dividends. These fluctuations in expectations are in turn caused by the release of new information. In an efficient market with constant expected returns, the price and return path can be demonstrated in the Figure 1 below.

Price and return changes are unpredictable, with price responding only to new information or news. For example, 'good' news about earnings prospects would cause the price to move from A to B. The return on the share is unpredictable and past returns cannot forecast future returns. Simply put, any information available at time  $t$  is of no use in predicting expected returns in future periods.

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<sup>2</sup>Rational expectations holds for all investors; dividend growth is not explosive and the terminal condition holds; and all investors have the same view of the determinants of returns and have homogeneous expectations.



The second type of model identified was the non-negative expected return model. It is based on an expectations model of share prices called a martingale with drift. Formally, if the expected value of the log of share prices in the next period is equivalent to the log of the current period's price plus a drift term:

$$E(\ln P_{t+1} / \phi_t) = \ln p_t + \delta \quad (11)$$

and as expected continuously compounded returns are equivalent to the difference between the log of expected and actual prices over that period:

$$E(R_{t+1} / \phi_t) = E(\ln p_{t+1} / \phi_t) - \ln p_t \quad (12)$$

then expected returns are equivalent to the drift term:

$$\begin{aligned} E(R_{t+1} / \phi_t) &= E(\ln p_{t+1} / \phi_t) - \ln p_t \\ &= \delta. \end{aligned} \quad (13)$$

The implication of the martingale with drift is that the set of "one security and cash" mechanical trading rules which concentrate on individual securities and state when a trader should sell, buy and hold a share cannot have greater expected returns than the strategy of simply buying and selling securities in the normal process of investment diversification.

The essential difference between the constant expected returns model and the non-negative expected returns model used to test the weak form of the EMH is that the random walk incorporates a stronger restriction on prices than the martingale (with drift)<sup>3</sup>. The random walk with drift rules out any linear or non-linear dependence amongst the error terms, whilst the martingale with drift only restricts the error terms to be uncorrelated. Furthermore, the martingale does not restrict the higher conditional moments such as the variance to be statistically independent as does the

<sup>3</sup> In a random walk with drift, the value of the drift term may be positive or negative, whereas in a martingale with drift, the drift is always positive.

random walk with drift. The martingale with drift thereby allows the conditional variance of price changes to be predictable from past information.

Note that these two models of expected returns are time-series models that examine multiple-period changes in prices, and so were used to empirically test the weak form of the EMH. As evidence in support of the Weak Form grew, empirical testing moved on to the stronger forms of the hypothesis. To achieve this, equilibrium expected return theories were developed to examine whether securities were efficiently priced in relation to one another. These models were cross-sectional, attempting to forward explanations of the causal relationships involved in the determination of equilibrium returns and asset prices in capital markets. In terms of the Semi-Strong Form of the EMH, the principal expected return theory used is the market model<sup>4</sup> suggested by Markowitz (1959). It says that the return on an asset is positively-related to the return on the market:

$$(r_{j,t} / \phi_t, r_{m,t}) = a_j + \beta_j r_{m,t} \quad (14)$$

where  $a_j$  and  $\beta_j$  are constants for asset  $j$  in period  $t$ . Thus, the expected returns on an asset reflects information that becomes available in the current period that to an extent affects the return on all securities. This theory has been used by such researchers as Fama, Fisher, Jensen and Roll (1969) to test the reaction of stocks to the announcement of such fundamental information as stock splits and dividend announcements.

The final expected return theory, used to test the Strong Form of the EMH, is the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965a, 1965b). It is the dominant theory of capital asset pricing in finance, with strong empirical support. The CAPM states that the expected return on a share is positively-related to the risk of the security:

$$E(R_{j,t+1} / \phi_t) = r_{f,t+1} + \beta_j [E(R_{m,t+1} / \phi_t) - r_{f,t+1}] \quad (15)$$

where  $\beta_j$  is the expected relative measure of risk for asset  $j$  in period  $t+1$ . From the CAPM, Jensen (1969) derived what has become known as the Jensen Index. It indicates the level of abnormal returns (or losses) that a portfolio has achieved. Formally:

$$J_j = E(R_{j,t+1} / \phi_t) - \{r_{f,t+1} + \beta_j [E(R_{m,t+1} / \phi_t) - r_{f,t+1}]\}. \quad (16)$$

Jensen (1969) used this index to assess the performance of mutual funds, which could be expected to have possible access to inside information, and thereby earn excess profits.

### **Empirical Testing of the Weak Form of the Efficient Markets Hypothesis**

In the early literature, there were three dominant tests of the Weak Form of the EMH. As mentioned previously, any test of the EMH is a joint test of the expected return model and the 'fair game' property of the EMH.

Joint tests of the constant expected returns/'fair game' property of the EMH have been conducted by calculating the serial correlations of successive price changes. Fama (1976) discusses the testing procedure. The test can be described by the regression equation:

<sup>4</sup> The market model has been referred to as the stochastic analogue of the CAPM; however, the two approaches have been identified separately here. This has been done to emphasise the fact that the former theory deals with realised values whereas the latter is based entirely upon expectations.

$$E(R_t / r_{t-\tau}) = \delta_\tau + \gamma_\tau r_{t-\tau} \quad (17)$$

where  $\tau$  is the lag,  $\gamma_\tau$  is the autoregressive coefficient and is equivalent to  $[cov(r_t, r_{t-\tau})] / \sigma^2(r_{t-\tau})$  and  $\delta_\tau$  is a constant. Assuming that the statistical process generating the returns is stationary through time so that the standard deviation of returns is constant on an asset:

$$\sigma(r_t) = \sigma(r_{t-\tau}) = \sigma(r_t) \quad (18)$$

then, the autoregressive coefficient  $\gamma_\tau$  and the autocorrelation coefficient for lag  $\tau$  are the same:

$$\begin{aligned} \rho(r_t r_{t-\tau}) &= cov(r_t r_{t-\tau}) / [\sigma(r_t) \sigma(r_{t-\tau})] \\ &= cov(r_t r_{t-\tau}) / [\sigma^2(r_{t-\tau})] = \gamma_\tau. \end{aligned} \quad (19)$$

If the market is efficient and equilibrium expected returns are constant, the estimated regression will show an autoregressive coefficient statistically equal to zero. This will thereby make expected returns statistically equal to a constant ( $\delta_\tau$ ). If the autocorrelation coefficient is not zero, the regression results indicate that either the market is inefficient so that some component of stock price changes can help predict future changes, or equilibrium expected returns are not constant.

Typical of tests conducted of this type was by Fama (1965). Firstly, he calculated sample serial correlations for daily changes in the log of price for lags 1 to 30 for each of the thirty stocks in the Dow Jones Industrial Index from 1957 to 1962. Secondly, sample serial correlations for lags 1 to 10 were computed for non-overlapping differencing intervals of 4, 9 and 16 days. In both cases, there was a finding of unsubstantial linear dependence among lagged daily stock price changes. In absolute terms, the coefficients are always close to 0.0. Note that although the correlation is not substantial, it is statistically significant, but in economic terms, most likely that it could not form the basis of a profitable trading system. Thus, although the constant expected returns model may have been statistically rejected, the serial correlation is so small that it provides a close approximation to reality. The efficient markets 'fair game' property had not been violated by these findings because the expectation of excess profits still remained zero. Noteworthy of Fama's (1965) research was that when examining the random walk model of stock price changes, he found evidence in support of Kendall's (1953) finding of a departure from normality in terms of leptokurtosis.

Another test of independence was of observing the randomness of share price changes from transaction to transaction. This test has become known as the Runs Test. A run is defined as a sequence of price changes of the same sign. In comparison, a reversal is a pair of consecutive price changes of opposite sign. Fama (1965) noted that large changes in daily prices follow large daily price changes, but with the signs of successive changes random. Researchers using this method have found two departures from complete randomness. Firstly, reversals are two to three times as likely as runs, and secondly, a run is slightly more frequent after a proceeding run than after a reversal. Once again note the significant departures from independence, but as previously mentioned, a denial of randomness does not in itself constitute a rejection of the EMH.

As Fama (1965) notes, from a practical viewpoint, the serial correlations and runs tests are too unsophisticated to detect the complicated patterns in a historical price sequence observed and supposedly exploited by chartists. Also, from a statistical viewpoint, both of these tests seek dependence which is present through all the data,

whilst it is possible that price changes are dependent only in special circumstances. Furthermore, note that the inference from both of these findings was that mechanical trading rules could not beat the buy-and-hold strategy. To test whether trading rules were actually inferior to the buy-and-hold strategy, filter tests were devised. Of the most prominent in this line of research is Alexander (1961, 1964) and Fama and Blume (1966). The filter test is essentially a test of the non-negative expected returns/‘fair game’ property of the EMH. An  $x\%$  filter rule is defined as follows: if the price moves up  $x\%$  from a trough, buy and hold the security until it moves down by  $x\%$  from a peak. At this point, sell the security and go short until it moves up by  $x\%$  from a subsequent trough. Thus, all moves of less than  $x\%$  are ignored. The idea here is that the  $x\%$  filter filters out all movements smaller than  $x\%$ , thereby removing the price volatility that may disguise the underlying trend. If there is an underlying trend in the data, investors would expect some excess profits from the filter rules. The conclusion of Fama and Blume (1966) and Alexander (1961, 1964) that various ‘one security and cash’ trading filters could not beat the buy-and-hold strategy is support of the ‘fair game’ property of the EMH.<sup>5</sup> But once again, note that it was possible to find evidence that is inconsistent with the non-negative expected returns model as the results from very small filters indicate that it is possible to devise trading systems based on intra-day price swings which could beat the buy-and-hold strategy. This was consistent with evidence for slight linear dependence in successive daily price changes. The average profits from the profitable trading filters were so small though that transaction costs would absorb the gains, causing the advantage over the buy-and-hold strategy to disappear.

In summary, the early literature contained powerful evidence in favour of market efficiency. Although noting departures from the random walk with drift and martingale with drift models of stock price movements, it was concluded that in economic terms these deviations were insignificant, so that the ‘fair game’ property of the EMH was not violated; thus, the EMH provided a strong representation of reality. It has been used to support two different conclusions. Firstly, portfolio managers could not outperform the market to a large extent by trading on publicly available information. Secondly, the hypothesis is often viewed as establishing that financial market prices represent rational assessments of fundamental values.

#### **Anomalies in the Weak Form of the Efficient Markets Hypothesis**

In the 1970s, the EMH was so strong that Jensen (1978: 95) stated that ‘there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Markets Hypothesis’. Yet he believed that there was a coming mini-revolution in the field of finance similar to that described by Kuhn’s scientific revolution. This was due to the development of better data and more sophisticated econometric techniques, which meant that more inconsistencies were found in the empirical testing of theories than were found in the past.

In the 1980’s and 1990’s, evidence began to build that share prices contained predictable components. Fortune (1991), using over 2,700 daily observations between

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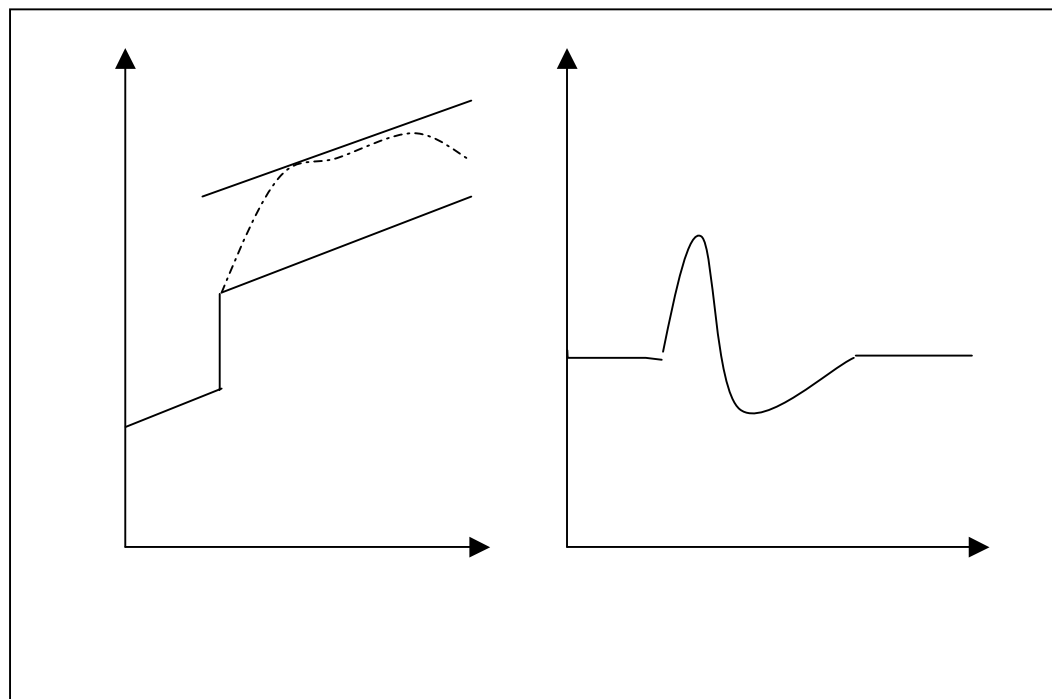
<sup>5</sup> It is to be noted that while the conclusion is correct, the risk involved in following a filter rule, where the investor moves in and out of securities is quite different to the risk involved in a buy-and-hold strategy so that the returns under the two systems are not comparable. Later research by Jensen and Bennington (1970) attempted to correct for this deficiency in controlling risk using the CAPM but again the control was not adequate to distinguish the strategies.

2 January 1980 and 21 September 1990 on the S&P 500 share index, tested the null hypothesis of constant expected returns by performing the following regression:

$$\begin{aligned}
 r_t &= \ln p_t - \ln p_{t-1} \\
 &= \alpha + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \theta_4 \varepsilon_{t-4} + \theta_5 \varepsilon_{t-5} \\
 &\quad + \beta_1 WE + \beta_2 HOL + \beta_3 JAN + \mu_t
 \end{aligned}
 \tag{20}$$

where  $\varepsilon_{t-i}$  are forecast errors;  $\theta_i$  are moving average coefficients;  $WE = 1$  if the trading day is a Monday and zero otherwise;  $HOL = 1$  if the current trading day precedes a one-day holiday and zero otherwise;  $JAN = 1$  for January trading days and zero otherwise; and  $\mu_t$  is the error term. It was found that  $MA(1)$ ,  $MA(4)$ ,  $MA(5)$  and  $WE$  were statistically significant, although the size of the adjusted R-squared ( $\bar{R}^2 = 0.01$ ) suggested that the potential to obtain abnormal profits on the information contained in these findings would involve substantial risk, with the very high transaction costs involved in trading the S&P 500 index outweighing any profits received. This finding was therefore a violation of the rational expectations element of the EMH under the null of constant expected returns, but it did not violate the criterion of being unable to persistently earn abnormal profits.

In contrast to the analysis of daily behaviour of share returns conducted by Fortune (1991), Fama and French (1988) test the hypothesis that over the long run, share returns display mean-reverting behaviour. Mean-reverting behaviour is demonstrated in Figure 2 below.



In comparison to the random walk with drift in share prices/constant expected returns model mentioned previously, the price overreacts to the fundamental information, rising past its 'intrinsic' value, and reaching a peak at Y. The price then moves back towards this intrinsic value. Prices are thereby said to be mean reverting. Note the return behaviour. Over short horizons, returns are positively serially correlated, with positive returns (from P to Q) followed by positive returns (from Q to R), and negative returns (from R to S) followed by negative returns (from S to T). Over long-horizons, returns are negatively serially correlated, so that a movement in one

direction (between P and R) is followed in the long run by movement in the opposite direction (between R and T, or Y).

To examine the extent of mean reversion in share returns, Fama and French (1988) separate the natural logarithm of share prices at time  $t$  ( $\ln p_t$ ) into a permanent or fundamental component and a transitory component. The fundamental or permanent component was modelled, as in the early literature, as a random walk with drift ( $q_t$ ), whilst the transitory component was modelled as a first-order Markov scheme ( $z_t$ ). Formally:

$$\begin{aligned}\ln p_t &= q_t + z_t \\ q_t &= q_{t-1} + \mu + \eta_t \\ z_t &= \phi z_{t-1} + \varepsilon_t\end{aligned}\tag{21}$$

where  $\eta_t$  and  $\varepsilon_t$  are white noise and  $\phi$  is the first-order autocorrelation coefficient. Since  $\ln p_t$  is the natural log of the stock price, the continuously compounded return from  $t$  to  $t+k$  is:

$$\begin{aligned}r(t, t+k) &= \ln p_{t+k} - \ln p_t \\ &= [q_{t+k} - q_t] + [z_{t+k} - z_t]\end{aligned}\tag{22}$$

The random walk component produces white noise in returns, so that it is possible to calculate the continuously compounded return from  $t$  to  $t+k$  produced by the stationary component of stock prices. The regression of  $(z_{t+k} - z_t)$  on  $(z_t - z_{t-k})$  will produce the autocorrelation coefficient of the stationary component. This methodology is calculated on monthly returns on equal and value-weighted portfolios of all NYSE stocks for the 1926-85 period. The portfolios are divided into industry and decile categories. Fama and French (1988) find that autocorrelations have a U-shape over time. This was consistent with their hypothesis that the stationary component would cause the autoregressive coefficient to approach  $-0.5$ , but that the greater variability of the random-walk component over the long return horizon would result in the non-stationary component dominating the stationary component. This would in turn force the autoregressive component to approach zero. Their findings supported this hypothesis. Autocorrelations became negative for two-year returns, reached a minimum for three-to-five year returns, and then moved back towards zero for longer return horizons. Predictable variation (obtained from the adjusted R-squared) is estimated by Fama and French (1988) to be about 40 percent of three-to-five year return variances for portfolios of small firms. The percentage falls to around 25 per cent for portfolios of large firms.

In contrast to Fama and French (1988), Poterba and Summers (1988) examined the variances of returns over different horizons to determine the strength of the mean-reversion in returns. If share returns are random, then variances of holding period returns ( $\sigma^2$ ) should increase in proportion to the return horizon ( $k$ ). Formally:

$$\text{var}(r_{t+1}^k) = k\sigma^2.\tag{23}$$

The Variance Ratio used by Poterba and Summers (1988) incorporates returns at different horizons, and is defined as:

$$VR(k) = (12/k) * \text{var}(r_t^k) / \text{var}(r_t^{12})\tag{24}$$

where  $r_t$  are monthly returns and  $r_t^k = \sum_{i=0}^{k-1} r_{t-i}$ . If this statistic converges to unity, then returns are uncorrelated through time. If some variation is due to transitory factors, then the variance ratio will be either above or below one. Negative autocorrelation at

some lags is indicated by a variance ratio below one. Alternatively, if the variance ratio rises above one, this will indicate positive autocorrelations at some lags. Poterba and Summers (1988) found that after analysing data on equal-weighted and value-weighted NYSE returns from the period 1926-1985, long-horizon stock returns have large predictable components, with point estimates implying that transitory components account for more than half of the monthly return variance. Furthermore, they found that share returns showed positive serial correlation over short periods of less than a year and negative serial correlation over longer return horizons of between three and eight years. This result was supported by data from other nations and time periods. Once again, the finding of a significant transitory return component and therefore mean-reverting behaviour provided supporting evidence on the predicability of share returns and a rejection of the constant expected returns model.

In 1991, Kim, Nelson and Startz reevaluated the empirical evidence relating to mean-reversion. By using the autoregressions discussed by Fama and French (1988) and the variance ratio methodology of Poterba and Summers (1988), they found, in contrast to these researchers, that the mean-reverting behaviour of share prices was a pre-war phenomenon, with post-war data actually displaying mean-averting behaviour. This result was obtained by using randomisation methods to calculate significance levels instead of the Monte Carlo method under a Normal assumption. But Cutler, Poterba and Summers (1991) provided evidence supporting the conclusion of Poterba and Summers (1988) and Fama and French (1988) using an array of data sets on asset types. They found that returns tend to be negatively serially correlated over long horizons but positively correlated over short horizons. Note that although the Variance Ratio and Regression tests assume linearity, the findings suggest the possibility of non-linear patterns in returns. An interesting approach that addresses the non-linearity was undertaken by McQueen and Thorley (1991). They used a Markov chain model to test the random walk hypothesis of share price returns. Returns are assumed to be drawn from either a high return state or a low return state, with the probability of staying in or switching to an opposite state governed by the transitional probabilities. This approach allows the parameters (transition probabilities) to vary depending on a given sequence of prior states. The random walk hypothesis imposes the constraint of equal transitional probabilities. After conducting likelihood ratio tests, the restriction is rejected, with the researchers finding that shares do not follow a random walk in the postwar period.

### **Interpretations of the Transitory Price Component**

As with any rejection of a joint test of the constant expected returns/‘fair game’ property of the EMH, the rejection can be attributed to either an incorrectly specified equilibrium expected returns model or to market inefficiency. Both Poterba and Summers (1988) and Fama and French (1988) suggest that the findings of a transitory price component are consistent with models of both market efficiency and market inefficiency.

The efficient markets explanation forwarded by Fama and French (1988) reads as follows: Consider the rational valuation formula (Cuthbertson (1996)) used by market practitioners in an efficient market to value a stock:

$$p_t = E \left[ \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1 + \delta_t)^i} \middle| \phi_t \right]. \quad (25)$$

Note that the required or expected return  $\delta_t$  now has a subscript, indicating time-varying expected returns. Now suppose that shocks to expected dividends and shocks to expected returns in any time period are independent, so that a shock to expected returns has no effect on the rational forecast of future dividends. Thus, a shock to expected returns in the current period is exactly offset by an opposite adjustment in the stock price in the current period. This, in turn, implies that the pattern of mean-reverting stock price components found in the data could well be explained by the efficient market's stock valuation model suggested above which incorporates time-varying expected returns.

Noting that serial correlation does not necessarily imply a violation of the EMH, Cecchetti, Mark and Lam (1990) investigated whether the pattern of mean reversion in long-run share returns could be explained by an equilibrium model of asset pricing. They used the Lucas (1978) Asset Pricing Model, which is a specific parameterisation of the consumption-CAPM, with the Hamilton (1989) model incorporated to describe in the model the time-series nature of consumption, dividends and output. They found that the commonly used measures of mean reversion in share prices calculated from historical returns data nearly always lie within a 60 percent confidence interval of the median of the Monte Carlo distributions implied by their equilibrium pricing model.

In contrast to the theories in support of market efficiency in the presence of mean reversion, the finding of a transitory share price component is also consistent with models of an inefficient market. In this hypothesis, a share price deviates from its fundamental or permanent component over a period of time before returning to its fundamental value. The predominant theory in the finance literature in this line of research is of a market segmented into smart money and noise traders. This contrasting theory to the EMH is summarised by Shleifer and Summers (1990). The smart money prices a share in accordance with its fundamental value, and makes rational buy/sell decisions accordingly. In an efficient market, risk-free arbitrage by investors ensures that prices are in line with fundamentals, but two types of risk in the real world limit the efficiency that risk-free arbitrage creates. Firstly, fundamental risk is the risk that the smart money believed that a share was incorrectly priced, but realised dividends actually equate to the expected value of future dividends, thereby eliminating arbitrage profits. Secondly, the risk that shares may actually move further away from their fundamental value can effect the degree of arbitrage in an inefficient market. Furthermore, in an efficient market it is assumed that traders actually know the fundamental value of a security. In the real world, traders may not know the fundamental value, or be able to detect deviations from it. This, in turn, makes arbitrage even more risky than before. Summers (1986), for example, suggested that an arbitrageur would have as hard a time as an econometrician in detecting deviations of price from its fundamental value. In contrast to the smart money, noise traders are irrational in their assessment of a share's value. Thus, if good news is released, the smart money will incorporate this information into their valuation of a share's prospects, pushing the price up. In turn, noise traders will see the share price rising, and believing it to be on an upward trend, purchase the share, thereby injecting their demand into the market for the share and pushing its price past its fundamental value. Therefore, prices move in response to changes in demand as well as changes in fundamental value. Arbitrageurs counter the shifts in demand prompted by the noise traders, but do not eliminate their effects on price completely. Thus, in a market comprised of smart money and noise traders, prices vary more than is warranted by changing fundamentals, since they respond to changing sentiment as well as news.

### The Markov, Regime-Switching, Non-Linear Econometric Model

In 1989, James Hamilton made a substantial contribution to non-stationary time series analysis by proposing a very tractable method in which to model occasional, abrupt shifts in the parameters governing the behavior of variables included in an econometric model. Many economic time series undergo episodes in which the behavior of the time-series seems to change quite dramatically. For example, the business cycle moves between expansionary and contractionary phases, and the sharemarket switches from bear to bull markets. The problem with such a data sequence is that the parameters governing a linear econometric model of the series do not change to account for the different behavior in the data. They therefore do not provide an optimal description of the behavior of the time-series. Furthermore, models that attempt to account for a historical change in the data from one phase to another through the inclusion of dummy variables do nothing for the model in terms of forecasting. Thus, what is required is a model which allows the parameter estimates to change in response to phase changes, whilst also providing the ability to forecast phases that the time-series will be in during future periods. Goldfeld and Quandt (1973) and Cosslett and Lee (1985) developed the Markov-switching regression model and Hamilton (1989) pioneered the Markov-switching, time-series version of this model incorporating autoregressive coefficients.

The essence of the Hamilton (1989) model is that sample observations are assumed to be drawn from two different distributions or states, with different parameters for each state determining the likelihood of observed data. The process governing the change from one phase or state to another is an unobserved random variable,  $S_t$ , described by a first-order Markov process, whereby the state of the system depends only on the state of the system in the previous period:

$$P(S_t = j / S_{t-1} = i) = p_{ij} \quad (26)$$

where  $j$  and  $i$  denote the state in the current and previous period respectively. Assuming  $N$ -states, these constant transition probabilities can be summarised in a  $N \times N$  matrix, denoted  $P$ :

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdot & p_{N1} \\ p_{22} & p_{22} & \cdot & p_{N2} \\ \cdot & \cdot & \cdot & \cdot \\ p_{1N} & p_{2N} & \cdot & p_{NN} \end{bmatrix} \quad (27)$$

The principal problem facing the econometrician in the estimation of the model is that the state of the system is unobservable. To solve this problem, Hamilton (1989) developed a recursive, non-linear filter from which was produced the sample conditional likelihood function and also probabilistic inferences about the state of the system. Hamilton (1989) applied this technique to the development of a two-state model of U.S. output which allowed the parameters of the model to change in response to the economy moving from a contractionary phase to an expansionary phase, or vice versa. In particular, the model suggested was an AR(4) model with a regime-switching mean:

$$y_t - \alpha_{s_t} = \phi_1(y_{t-1} - \alpha_{s_{t-1}}) + \phi_2(y_{t-2} - \alpha_{s_{t-2}}) + \phi_3(y_{t-3} - \alpha_{s_{t-3}}) + \phi_4(y_{t-4} - \alpha_{s_{t-4}}) + \varepsilon_t \quad (28)$$

where  $y_t = \ln(GNP)_t - \ln(GNP)_{t-1}$ ,  $\alpha_{s_t}$  is the state-dependent mean,  $\phi$  is the first-order autoregressive coefficient for lags one through to four, and  $S_t$  takes on the value of

zero or one, depending on whether the economy was in a contractionary state ( $S_t = 1$ ) or an expansionary state ( $S_t = 0$ ) at time  $t$ . Let the parameter vector to be estimated by Maximum Likelihood Estimation (MLE), including transitional probabilities, be denoted by  $\theta$ :

$$\theta = (\alpha_0, \alpha_1, p, q, \phi_1, \phi_2, \phi_3, \phi_4, \sigma^2). \quad (29)$$

Note that the Hamilton (1989) model keeps the variance equal to deal with the singularity problem (which is discussed later), but in doing so, removes the ability of the model to possibly identify two regimes of differing volatility in the data. In subsequent extensions to his 1989 model, Hamilton also allows the variance to be state-dependent.

The filtering process assumes a set of values for the parameter vector and calculates in each step of the filter the sample likelihood of the data. In a particular iteration, given a set of parameter values, the filter takes as input for observation  $t$  the joint conditional probability:

$$P(S_{t-1} = s_{t-1}, \dots, S_{t-4} = s_{t-4} / y_{t-1}, \dots, y_{-3}; \theta) \quad (30)$$

with these  $2^4 = 16$  probabilities summing to unity by construction. For each period, the filter iterates through the following steps:

**Step One:** Calculate the forecast state probability ( $S_t, S_{t-4}$ ):

$$\begin{aligned} & P(S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_{t-1}, \dots, y_{-3}; \theta) \\ & = P(S_t = s_t / S_{t-1} = s_{t-1}) \\ & \times P(S_{t-1} = s_{t-1}, \dots, S_{t-4} = s_{t-4} / y_{t-1}, \dots, y_{-3}; \theta). \end{aligned} \quad (31)$$

**Step Two:** Calculate the joint conditional density of  $y_t$  and ( $S_t, S_{t-4}$ ):

$$\begin{aligned} & f(y_t, S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_{t-1}, \dots, y_{-3}; \theta) \\ & = f(y_t / S_t = s_t, \dots, S_{t-4} = s_{t-4}, y_{t-1}, \dots, y_{-3}; \theta) \\ & \times P(S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_{t-1}, \dots, y_{-3}; \theta) \end{aligned} \quad (32)$$

where, assuming conditional normality:

$$\begin{aligned} & f(y_t / S_t = s_t, \dots, S_{t-4} = s_{t-4}, y_{t-1}, \dots, y_{-3}; \theta) \\ & = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{ -\frac{((y_t - \mu_{s_t}) - \phi_1(y_{t-1} - \mu_{s_{t-1}}) - \dots - \phi_4(y_{t-4} - \mu_{s_{t-4}}))}{2\sigma^2} \right\}. \end{aligned} \quad (33)$$

**Step Three:** The unconditional density of  $y_t$  can then be calculated by iterating Step

Two over the two states:

$$\begin{aligned} & f(y_t / y_{t-1}, \dots, y_{-3}; \theta) \\ & = \sum_{S_t=0}^1 \dots \sum_{S_{t-4}=0}^1 f(y_t, S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_{t-1}, \dots, y_{-3}; \theta). \end{aligned} \quad (34)$$

**Step Four:** Now the probability of  $(S_t, \dots, S_{t-4})$  conditional on  $(y_t, \dots, y_{-3})$  can be calculated:

$$\begin{aligned} & P(S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_t, \dots, y_{-3}; \theta) \\ &= \frac{f(y_t, S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_t, \dots, y_{-3}; \theta)}{f(y_t / y_{t-1}, \dots, y_{-3}; \theta)}. \end{aligned} \quad (35)$$

**Step Five:** The desired output, being the  $2^4=16$  probabilities required as input

into next iteration, can be calculated as follows:

$$\begin{aligned} & P(S_t = s_t, \dots, S_{-3} = s_{-3} / y_t, \dots, y_{-3}; \theta) \\ &= \sum_{S_{t-4}=0}^1 P(S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_t, \dots, y_{-3}; \theta). \end{aligned} \quad (36)$$

To start the filter, we need an initial guess of the parameter vector and the initial state probabilities. Hamilton (1989) suggests starting the filter with the unconditional probability  $P(S_0 = s_0, S_{-1} = s_{-1}, \dots, S_{-3} = s_{-3})$  by setting  $P(S_{-3}=1)$  equal to the ergodic probability of the Markov process :

$$P(S_{-3} = 1; \theta) = \frac{(1 - p_{11})}{(2 - p_{00} - p_{11})} \quad (37)$$

and  $P(S_{-3} = 0; \theta) = (1 - P(S_{-3} = 1; \theta))$ . Then for  $\tau = -2, -1, \dots, 0$  calculate:

$$\begin{aligned} & P(S_\tau = s_\tau, \dots, S_{-3} = s_{-3}) \\ &= P(S_\tau = s_\tau / S_{\tau-1} = s_{\tau-1}) \times P(S_{\tau-1} = s_{\tau-1}, \dots, S_{-3} = s_{-3}). \end{aligned} \quad (38)$$

Alternatively, the starting probabilities could be included in the parameter vector as additional parameters to be estimated by maximum likelihood, or simply, the econometrician can make a guess as to the value of the initial starting probabilities and parameter vector (Hamilton (1990)).

One byproduct of the filter is the sample conditional log likelihood. It can be obtained by taking the natural logarithm of the unconditional density of  $Y_t$  from Step Three and summing over the entire sample period  $T$ :

$$\begin{aligned} \text{Log}L &= \log f(y_T, \dots, y_1 / y_0, \dots, y_{-3}; \theta) \\ &= \sum_{t=1}^T \log f(y_t / y_{t-1}, \dots, y_{-3}; \theta). \end{aligned} \quad (39)$$

This can be maximised numerically with respect to the parameter vector  $\theta$ , and the resulting estimate  $\hat{\theta}$  is the MLE of the population parameter vector  $\theta$ .

Another byproduct of the basic filter is the filtered inference about the state of the system at time  $t$ , which can be calculated from the output of iteration at time  $t$ . The filtered inference uses all information up until time  $t$  to calculate the probability of which state the system was in during period  $t$ . Formally:

$$\begin{aligned} & P(S_t = s_t / y_t, \dots, y_{-3}; \theta) \\ &= \sum_{S_{t-1}=0}^1 \dots \sum_{S_{-3}=0}^1 P(S_t = s_t, \dots, S_{t-4} = s_{t-4} / y_t, \dots, y_{-3}; \theta). \end{aligned} \quad (40)$$

The smoothed inference is the probability of being in a state at time  $t$  derived from the full sample of observations. The optimal smoothed inference is calculated using an algorithm developed by Kim (1994):

$$\begin{aligned}
& P(S_{t-3} = s_{t-3}, \dots, S_{t+1} = s_{t+1} / \phi_T) \\
&= \frac{P(S_{t-2} = s_{t-2}, \dots, S_{t+1} = s_{t+1} / \phi_T) \times P(S_{t-3} = s_{t-3}, \dots, S_t = s_t / \phi_t) \times P(S_{t+1} = s_{t+1} / S_t = s_t)}{P(S_{t-2} = s_{t-2}, \dots, S_{t+1} = s_{t+1} / \phi_t)}.
\end{aligned}
\tag{41}$$

In estimating a mixture of normal distributions, a global maximum for the log likelihood function does not exist because it is not unimodal<sup>6</sup>; therefore the econometrician must seek to find a local maximum of the log likelihood function.<sup>7</sup> Hamilton (1991) discusses two problems associated with this objective. Firstly, Hamilton (1991: 27) explains that ‘a singularity arises whenever one of the distributions is imputed to have a mean exactly equal to one of the observations with no variance’. At any such point, the log likelihood blows out to infinity. A second problem can occur in small samples if two observations happen to be very close. Hamilton (1991: 27) notes in this situation that:

The largest bounded local maximum could come from parameter estimates that associate these two points with a distribution with a tiny variance and that attribute the other data to more reasonably shaped distributions.

To solve these problems, Hamilton (1989) restricts the variances in both states to be equal. In a later paper, Hamilton (1991) suggests that in such a situation, a Quasi-Bayesian approach could be adopted to deal with these problems. Here, the suggestion is to take as an estimator  $\hat{\theta}$ , the value that maximises the following objective function:

$$\begin{aligned}
Q(\theta) &= \log f(y_T, \dots, y_1 / y_0, \dots, y_{-3}; \theta) - \sum_{j=1}^K (a_j / 2) \log(\sigma_j^2) \\
&\quad - \sum_{j=1}^K b_j / (2\sigma_j^2) - \sum_{j=1}^K c_j (m_j - \mu_j)^2 / (2\sigma_j^2)
\end{aligned}
\tag{42}$$

where  $K$  represents the total number of states,  $j$  represents a particular state,  $m_j$  and  $b_j/a_j$  represent the analyst’s prior expectations about  $\mu_j$  and  $\sigma_j^2$ , whilst the values for  $a_j$  and  $c_j$  represent the confidence the analysts has in these priors.

#### Extensions to the Hamilton (1989) Model

The Hamilton (1989) model proved very effective in both dating the phase changes in the U.S. business cycle in accordance with NBER dating, and also measuring the average duration of the cycle. It subsequently became a popular model for econometricians to use in analysing a variety of economic situations in which it could be assumed that the time-series undergoes fundamental changes. For example, Engel and Hamilton (1990) developed a stochastic, segmented trend model to analyse the behavior of the U.S. dollar. In this specification, no autoregressive parameters were used, thereby allowing the log of the exchange rate to follow a within-state random walk with drift. The functional form was:

$$y_t = \ln ER_t - \ln ER_{t-1} = \mu_{s_t} + \varepsilon_t
\tag{43}$$

<sup>6</sup> A unimodal log likelihood function occurs when there is a unique value  $\theta$  for which  $\partial L(\theta) / \partial \theta = 0$ .

<sup>7</sup> The difference between the mixture of normals and the regime-switching model is that draws of  $Y_t$  follow a Markov process in the regime-switching model, whereas in the ‘mixtures’ model, successive draws are independent of each other.

where  $y_t$  is the change in the natural logarithm of the exchange rate,  $\mu_{s_t}$  is the state-dependent mean rate of appreciation or depreciation in the exchange rate and  $\varepsilon_t$  is the normally distributed error term. They found that movements in the dollar appeared to be characterised by long swings, thereby rejecting the null of a random walk with drift in the log of the exchange rate.

Most of the studies that use an extension of the Hamilton (1989) model use constant probabilities of staying in the one state or switching between states. The regime-switching model though can be extended to allow exogenous factors to effect the transitional probabilities. This adds extra flexibility to the model by allowing the probability of switching between regimes to increase or decrease in accordance with changes in a desired indicator. A popular model used by econometricians to achieve this purpose is the logistic functional form which takes the information variables,  $Z_t$ , and forces the dependent variable, being the computed transitional probabilities, to lie in the 0 - 1 range. Thus, the state of the process not only depends on the state in the previous period, but also on the value of the information variable in the previous period:

$$P_t(S_t = j / S_{t-1} = i, Z_{t-1}) = p_{ijt} = \frac{1}{1 + \exp^{(-Z_{t-1}\beta_s)}} \quad (44)$$

where  $s = i$  or  $j$ ,  $Z_t = (1, z_{1t-1}, z_{2t-1}, \dots, z_{(k-1)t-1})$ ,  $\beta_s = (\beta_{s0}, \beta_{s1}, \dots, \beta_{s(k-1)})'$  and  $k$  is the number of information variables which determine the time-varying transitional probabilities. Equation (44) collapses to constant transitional probabilities if only the  $\beta_{s0}$  are non-zero. Filardo (1994), using a logistic function, found that U.S. Industrial Production was well modelled by a specification that allowed the transitional probabilities to be determined by fluctuations in the U.S. Composite Index of Eleven Leading Indicators (CLI).

The Markov, regime-switching model can also be extended to account for state-dependent ARCH and GARCH effects. Hamilton and Susmel (1994) used a regime-switching ARCH (SWARCH) specification to model discrete changes in the volatility of stock returns caused by switching between states. They found that the SWARCH model offers a better statistical fit to the data and superior forecasts than models from the traditional ARCH and GARCH families. Gray (1996) also made a significant contribution to the literature by developing a method in which to estimate a regime-switching GARCH (GRS) model. He used this specification to model short-term interest rates, and found that the short-rate exhibited a different degree of mean reversion and a different form of conditional heteroscedasticity in each regime.

Note that the method is not limited to a process consisting of only two states. Hamilton (1994) tested to see whether real interest rates were characterised by three states. He found that they were well modelled by a specification which identified three distinct regimes in real interest rates; those being of a very high positive rate, a normal real rate and a negative real interest rate.

Thompson, Lead and Smith (1998) and Lead (1997) looked at fitting the Hamilton model to the Australian All Ordinaries Index for the period from 5 January 1968 to 26 September 1997. This research found that the model had strong descriptive power; however this strength did not translate into forecasting ability. In relation to forecasting, the regime-switching model was dominated by a simple random walk with drift.

## **Conclusion**

While there is general acceptance that the market is weak form and strong form efficient, it has not been possible to model precisely the behaviour of share prices across time. It also seems reasonable to assume that the information that is going to influence price enters the market randomly. This pattern with the efficiency level of the market ensures a fundamental random model. The drift characteristic is a little more difficult to explain; however, it may simply indicate that values increase across time to reflect the return element that is driven by the choice to invest in a risky asset. It may be that the nature of share prices is such that they can never be constrained within mathematical models. While more sophisticated techniques, such as regime-switching or moving parameter estimation, may provide good within-sample description they will not be able to handle the complexities of prices in relation to forecasting.

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